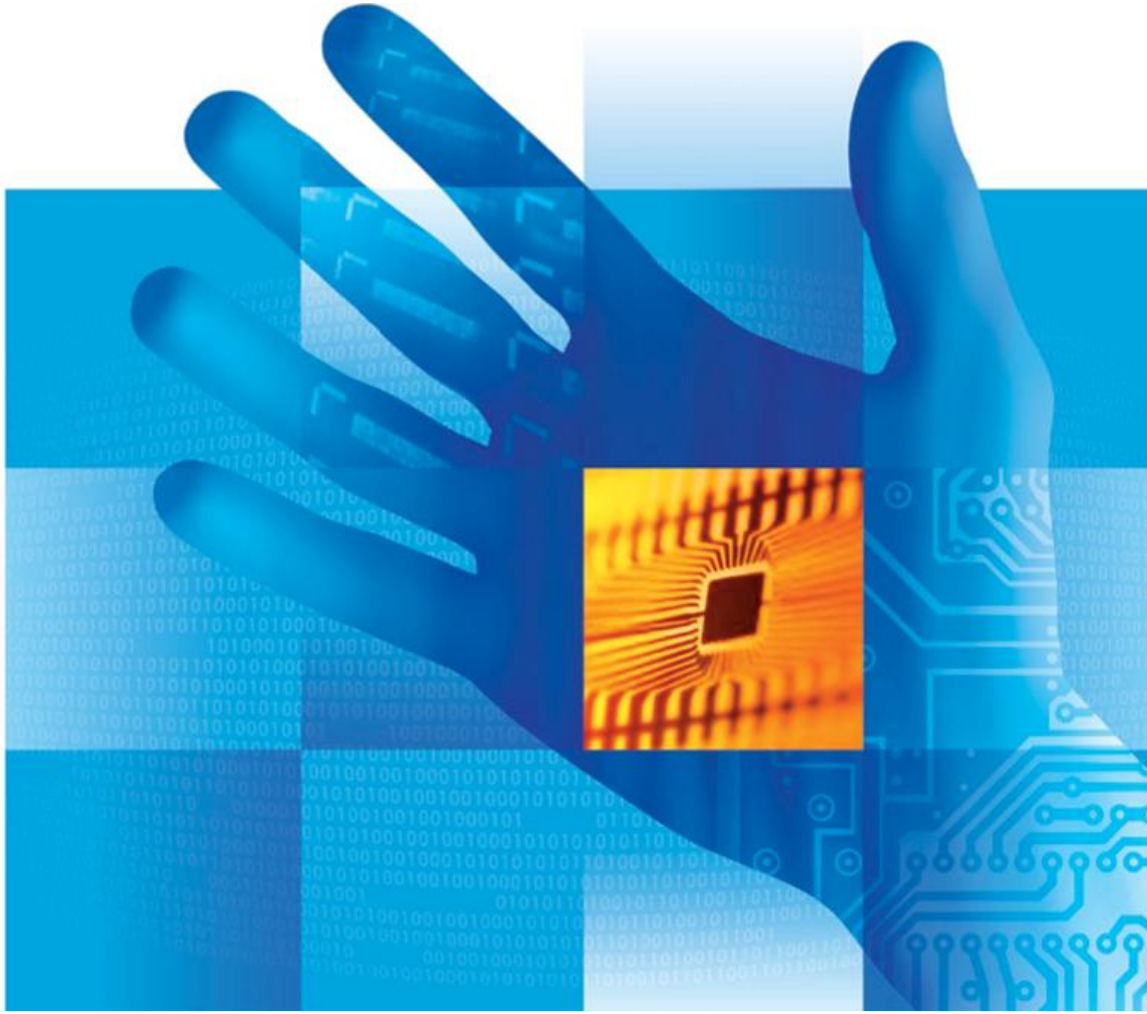
EVROPSKÁ UNIE
Evropské strukturální a investiční fondy
Operační program Výzkum, vývoj a vzdělávání

MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY

2. ELECTRICAL CIRCUITS



Doc. Ing. Stanislav Kocman, Ph.D.

20. 10. 2017, Ostrava

LECTURE PLAN

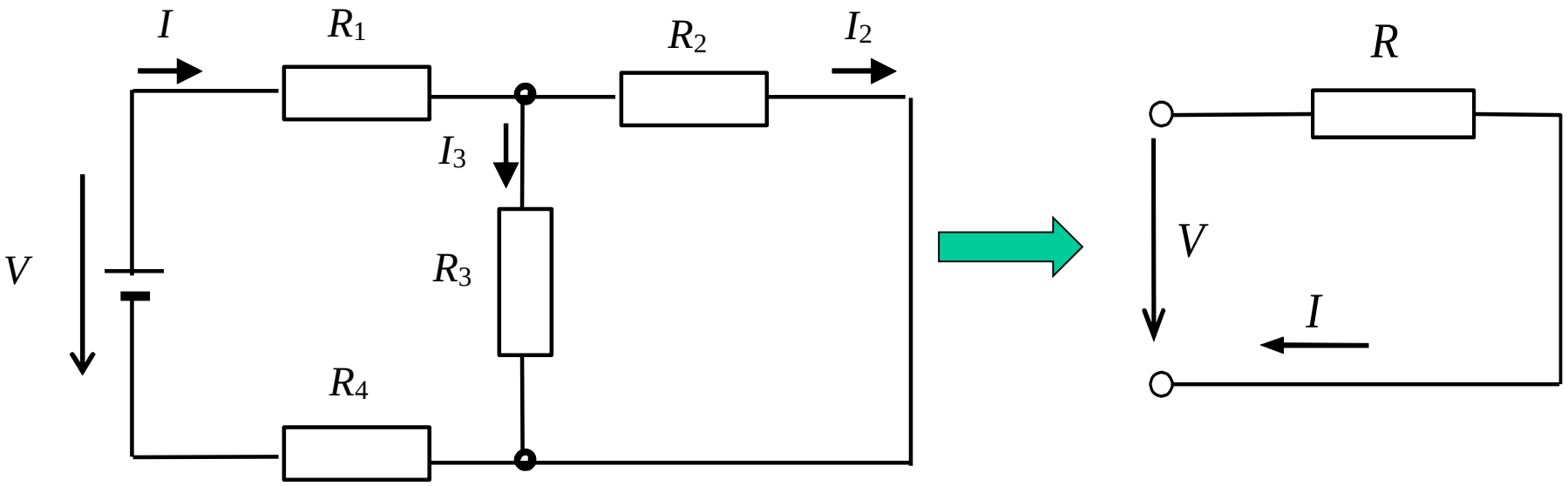
- methods of solving DC electrical circuits
- alternating circuits: basic definitions and quantities
- types of power of AC circuit and their relevance
- methods of solving AC electrical circuits

Methods of solving DC electrical circuits

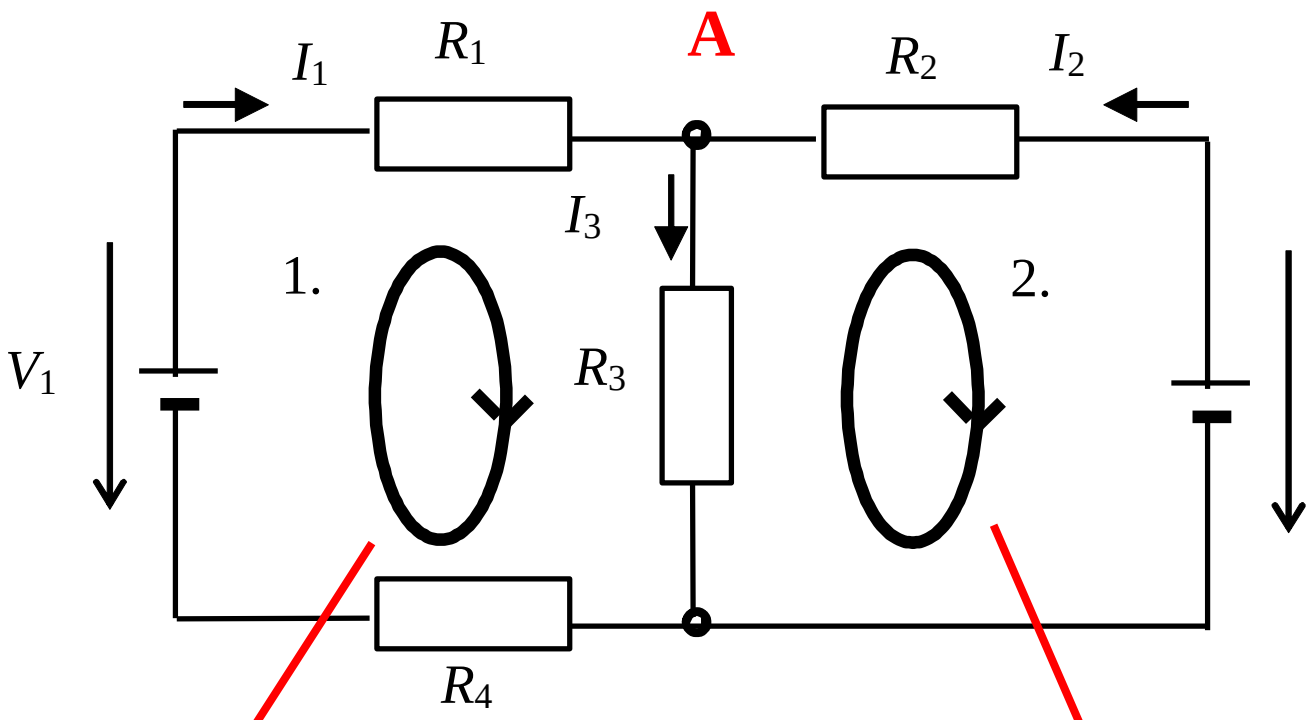
- **Methods overview**
  - ❑ step-by-step simplification
  - ❑ solution using Kirchhoff´s laws
  - ❑ the mesh current method
  - ❑ the node voltage method
  - ❑ the principle of superposition
  - ❑ Thévenin and Norton equivalent circuits

❑ Step-by-step simplification

- ➢ for single voltage source circuits
  - ➢ circuit replacement by one resistor and calculation of its resistance
    - ➢ resistors in series
    - ➢ resistors in parallel
    - ➢ delta-star transformation
  - ➢ calculation of total current
  - ➢ solution of specified voltages or currents



❑ Solution using Kirchhoff´s laws



node A

$$I_1 + I_2 - I_3 = 0$$

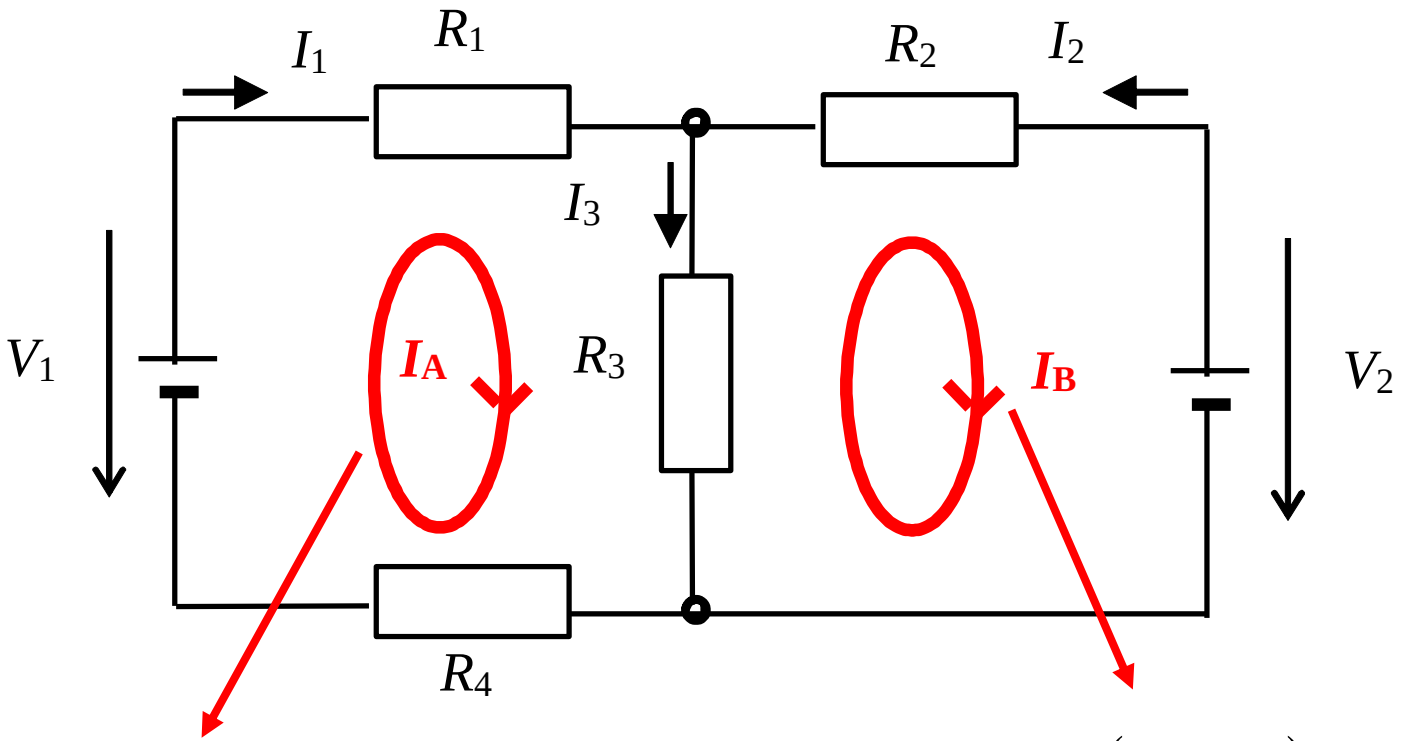
$$1. -V_1 + R_1 \cdot I_1 + R_3 \cdot I_3 + R_4 \cdot I_1 = 0$$

$$2.\ V_2 - R_3 \cdot I_3 - R_2 \cdot I_2 = 0$$

Three equations with three unknown currents

The mesh current method

- defining mesh currents (I_A, I_B,….)
- applying Kirchhoff voltage law for each independent mesh and calculating mesh currents
- solving the real currents

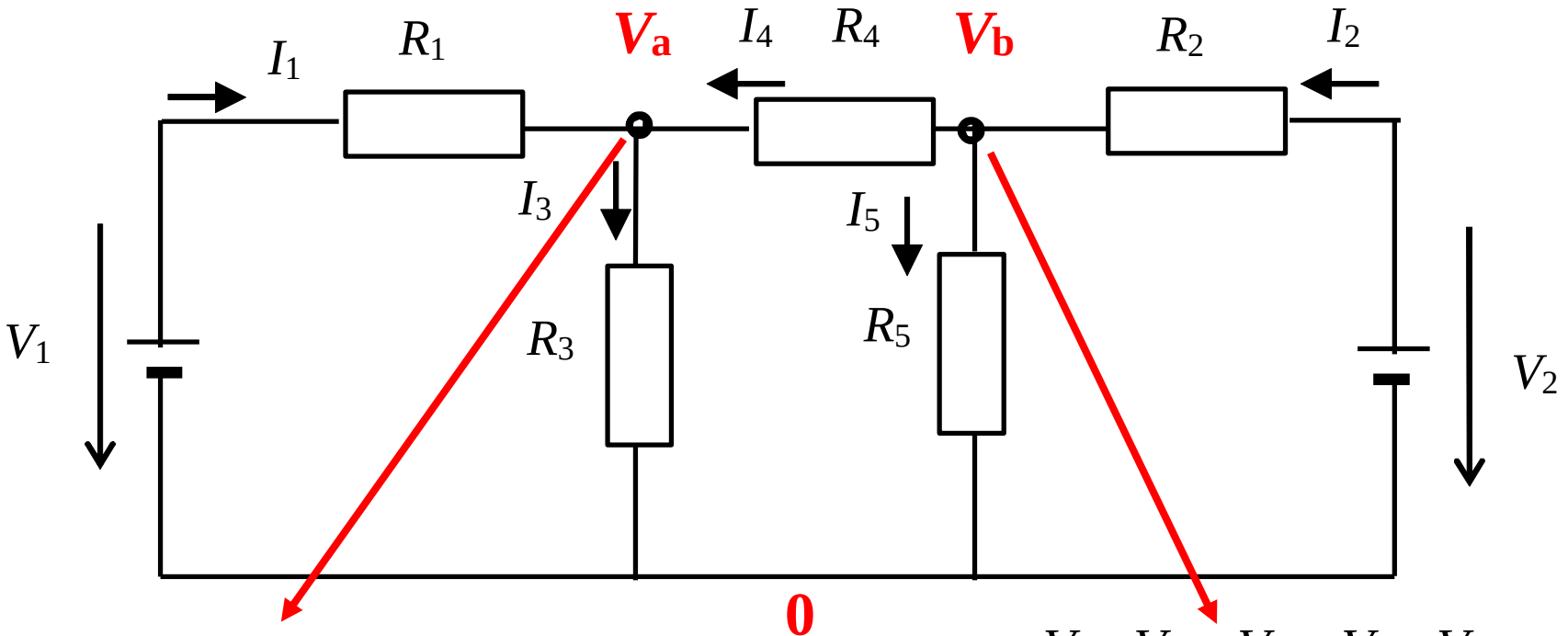


$$-V_1 + R_1 \cdot I_A + R_3 \cdot (I_A - I_B) + R_4 \cdot I_A = 0 \qquad V_2 + R_3 \cdot (I_B - I_A) + R_2 \cdot I_B = 0$$

$$I_1 = I_A,\ I_2 = -I_B,\ I_3 = I_A - I_B$$

The node voltage method

- selection of a reference node (0)
- defining node voltages (V_A, V_B…)
- applying Kirchhoff current law at each node
- solving the real currents



$$\frac{V_1 - V_A}{R_1} + \frac{V_B - V_A}{R_4} - \frac{V_A}{R_3} = 0$$

$$\frac{V_2 - V_B}{R_2} - \frac{V_B}{R_5} - \frac{V_B - V_A}{R_4} = 0$$

$$I_3 = V_A / R_3, I_5 = V_B / R_5 .....$$

Electrical quantities: definitions and equations

Electrical current: $i = \frac{q}{t} \quad (\mathrm{A}, \mathrm{C}, \mathrm{s})$

Steady DC current: $I = \frac{Q}{t} \quad (\mathrm{A}, \mathrm{C}, \mathrm{s})$

Electrical current density: $J = \frac{I}{S} \quad (\frac{\mathrm{A}}{\mathrm{m}^2}, \mathrm{A}, \mathrm{m}^2)$

Electrical voltage: $V_{12} = V_1 - V_2 \quad (\mathrm{V})$

Electrical potential: $V = \frac{W}{Q} \quad (\mathrm{V}, \mathrm{J}, \mathrm{C})$

potential energy

Electrical conductivity: $G = \frac{1}{R} \quad (\mathrm{S})$

Power of DC current: $P = V \cdot I \quad (\mathrm{W}, \mathrm{V}, \mathrm{A})$

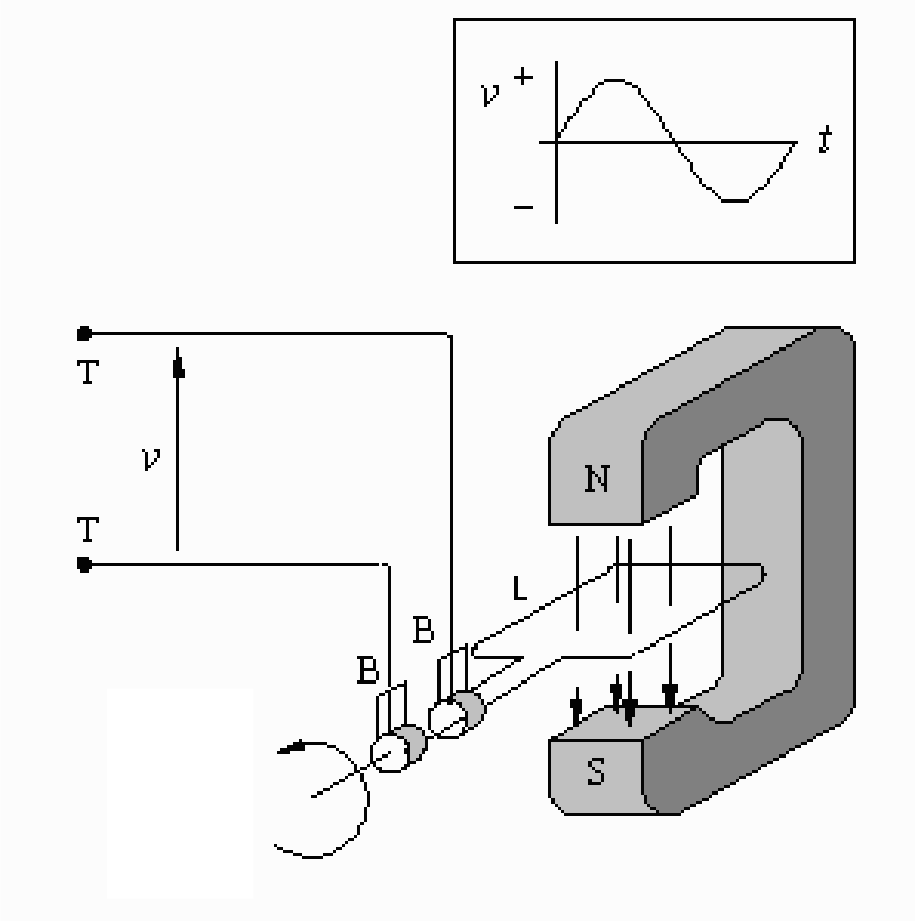
Energy of DC current: $W = V \cdot I \cdot t \quad (\mathrm{J}, \mathrm{V}, \mathrm{A}, \mathrm{s})$

Efficiency: $\eta = \frac{P}{P_1} \leq 1$

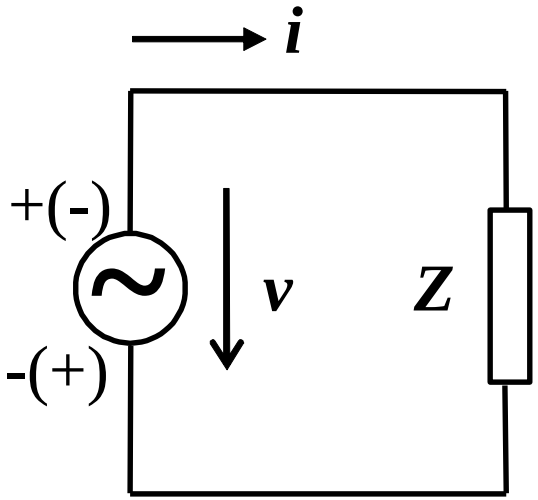
input

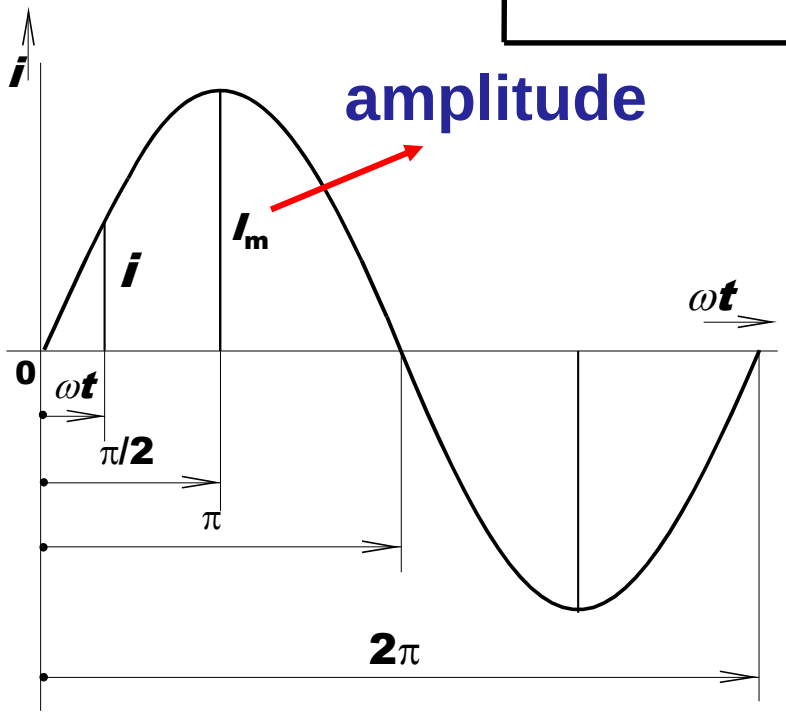
Alternating circuits: basic definitions and quantities

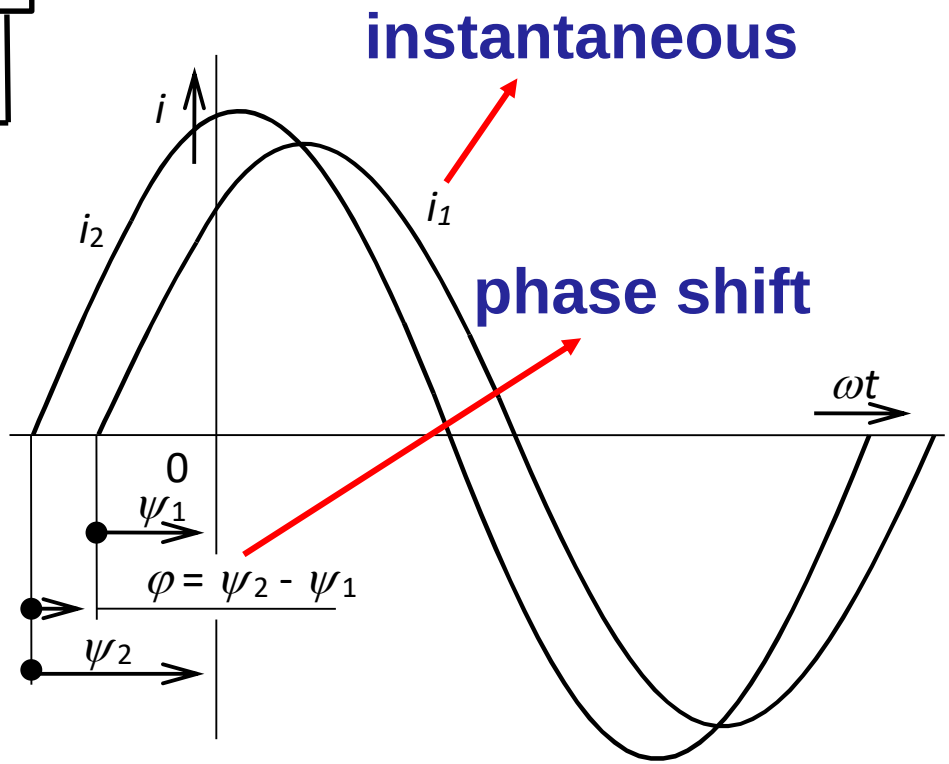
Alternating harmonic voltage



Alternating harmonic current



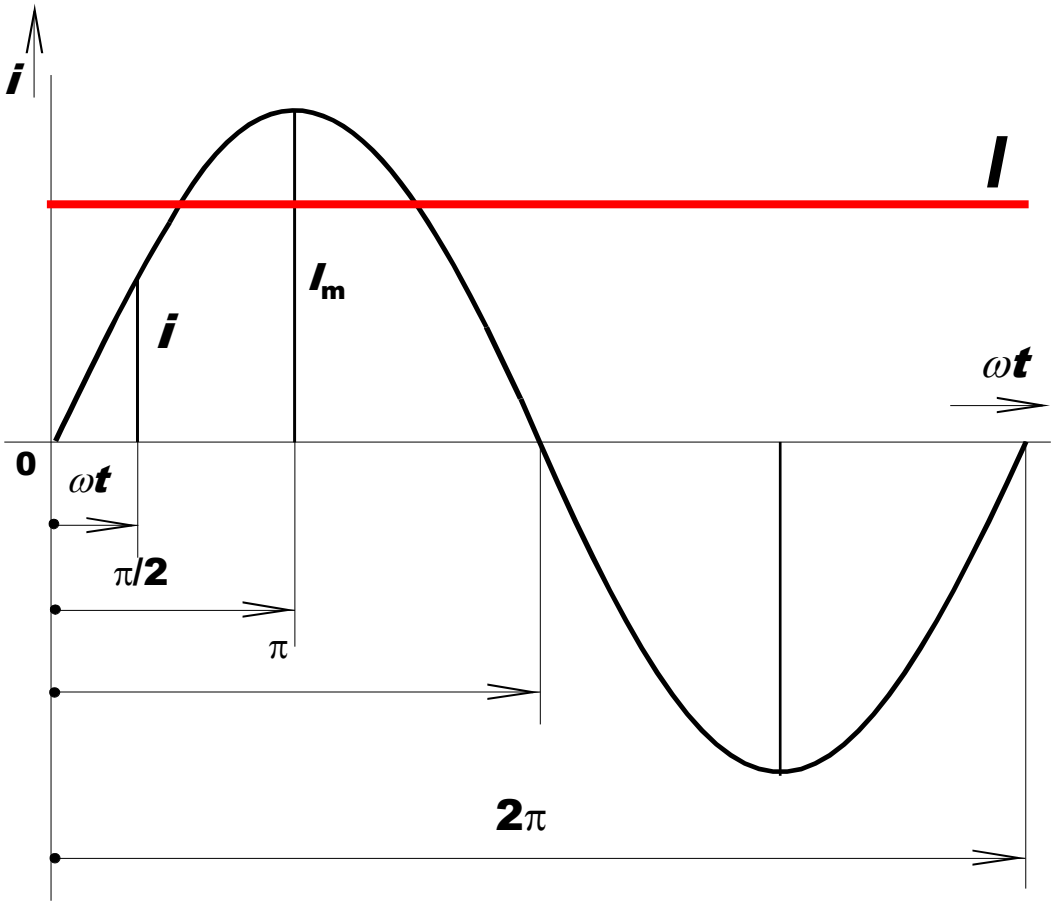




$$i = I_m \cdot \sin(\omega t) \quad \omega = 2\pi \cdot f = \frac{2\pi}{T}$$

$$i = I_m \cdot \sin(\omega t + \psi)$$

rms value



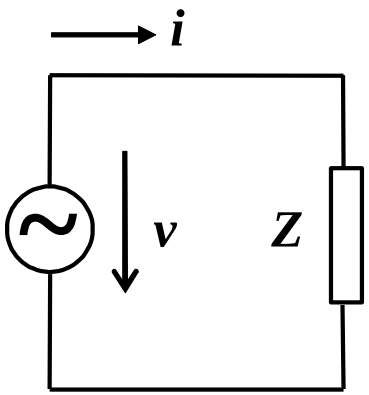
The rms value of the current *i*(t) is the DC current *I* that causes the same average power (or energy) to be dissipated by the resistor.

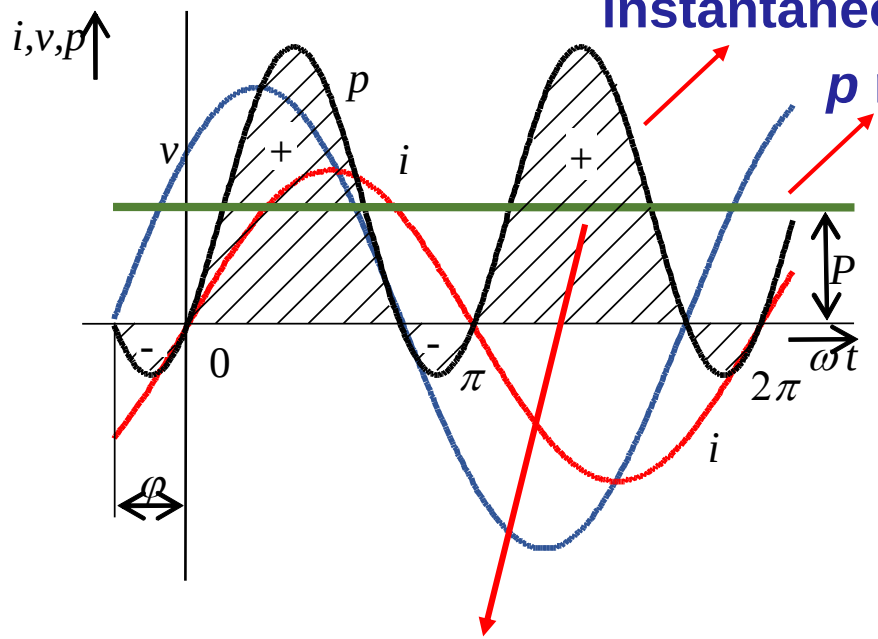
$$W = R \cdot I^2 \cdot T = \int_0^T R \cdot i^2 \cdot \mathrm{d}\, t$$

$$I = \sqrt{\frac{1}{T} \cdot \int_0^T i^2 \cdot \mathrm{d}\, t} = \sqrt{\frac{1}{T} \cdot \int_0^T {I_m}^2 \cdot \sin^2 \omega t \, \mathrm{d}\, t}$$

$$I = \frac{I_\mathrm{m}}{\sqrt{2}}$$

Types of power





instantaneous power *p*

***p* mean value**

$$v = V_m \cdot \sin(\omega t + \varphi),\ i = I_m \cdot \sin \omega t,\ p = v \cdot i \rightarrow V \cdot I \cdot \cos\varphi - V \cdot I \cdot \cos(2\omega t + \varphi)$$

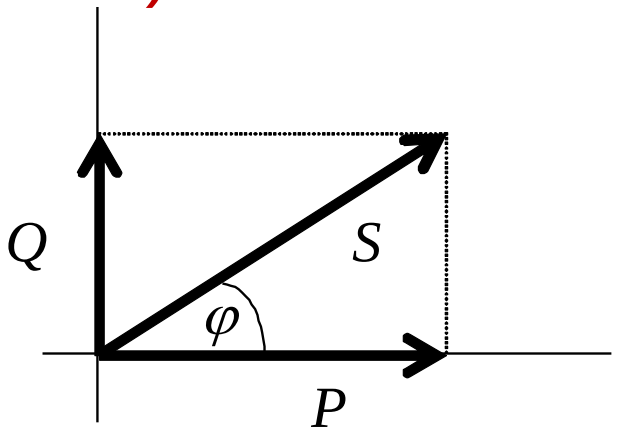
real power (mean value of instantaneous power)

$$P = V \cdot I \cdot \cos\varphi \ \ (\mathrm{W}),\ \ \cos\varphi = \mathrm{power\,factor}$$

reactive power $Q = V \cdot I \cdot \sin\varphi\,(\mathrm{var})$

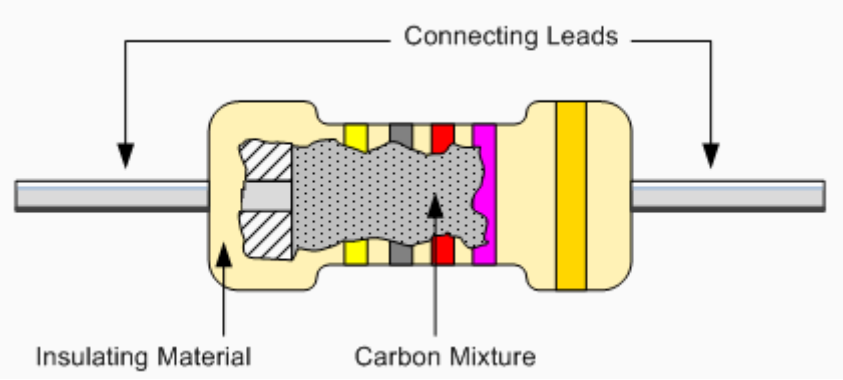
apparent power $S = V \cdot I \ \ (\mathrm{V \cdot A})$

$$S = \sqrt{P^2 + Q^2}$$

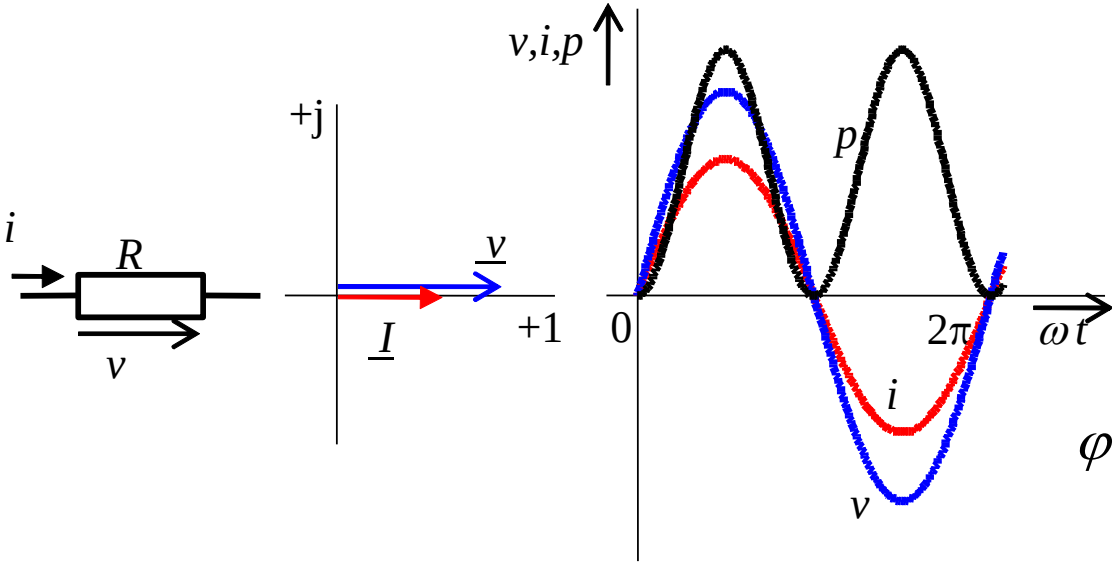


Methods of solving AC electrical circuits

Resistor in AC circuit







$$v = R \cdot i = R \cdot I_m \cdot \sin \omega t = V_m \cdot \sin \omega t$$

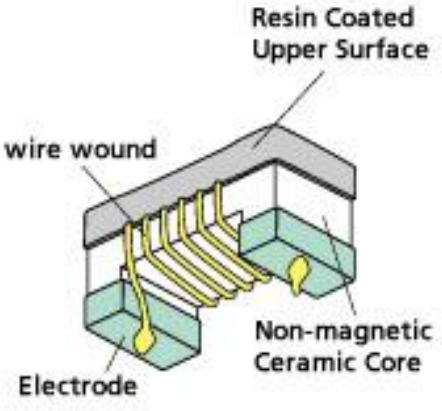
$$V_m = R \cdot I_m$$

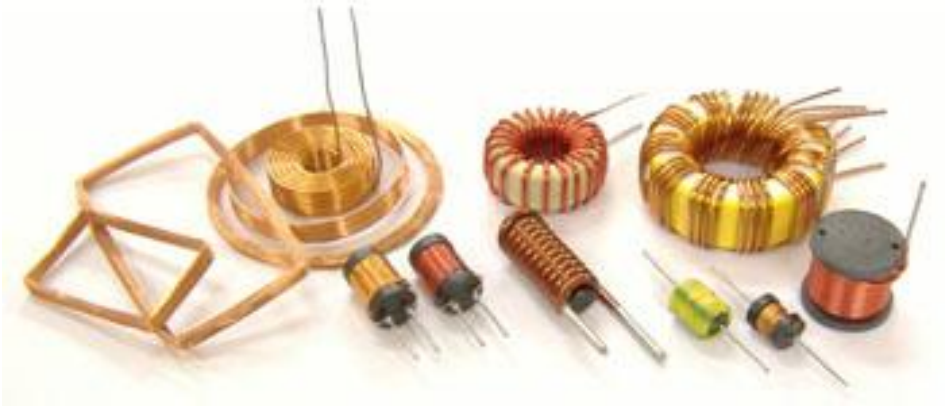
$$R = \frac{v}{i} = \frac{V}{I} \rightarrow V = R \cdot I$$

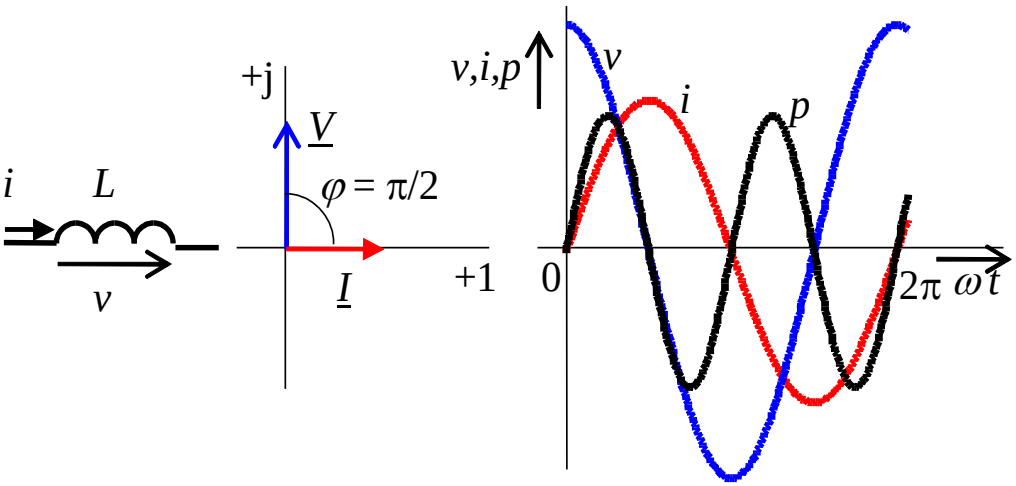
$$\varphi = 0 \rightarrow \cos \varphi = 1 \rightarrow P = V \cdot I = R \cdot I^2$$

Inductor in AC circuit









$$v = L \cdot \frac{\mathrm{d}\, i}{\mathrm{d}\, t} = V_{\mathrm{m}} \cdot \sin\left(\omega t + \frac{\pi}{2}\right)$$

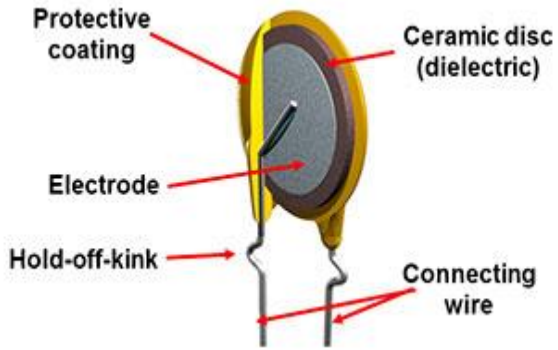
$$V_{\mathrm{m}} = \omega \cdot L \cdot I_{\mathrm{m}} = X_{\mathrm{L}} \cdot I_{\mathrm{m}} \rightarrow \; X_{\mathrm{L}} = \omega \cdot L\,(\Omega)$$

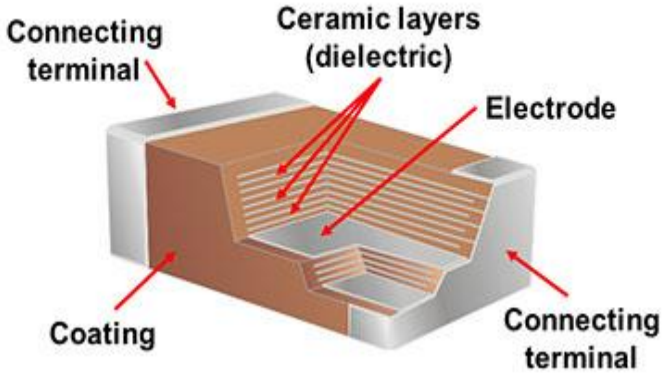
$$V = X_{\mathrm{L}} \cdot I$$

$$\varphi = 90^{0} \rightarrow \cos\varphi = 0 \rightarrow P = 0$$

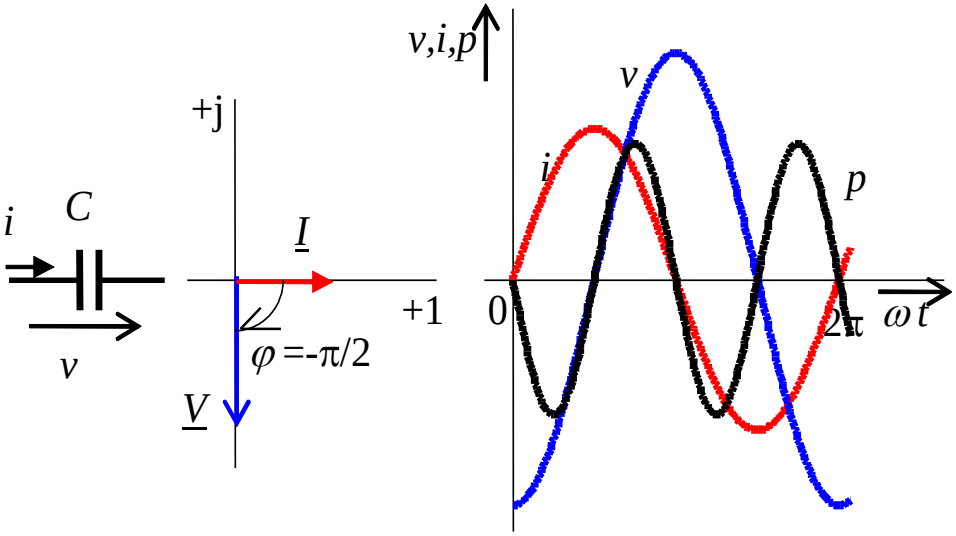
$$\sin\varphi = 1 \rightarrow \; Q = V \cdot I = X_{\mathrm{L}} \cdot I^{2}$$

Capacitor in AC circuit









$$i = C \cdot \frac{\mathrm{d}\,v}{\mathrm{d}\,t} \rightarrow v = \frac{1}{C}\int i \cdot \mathrm{d}\,t = V_{\mathrm{m}} \cdot \sin\left(\omega t - \frac{\pi}{2}\right)$$

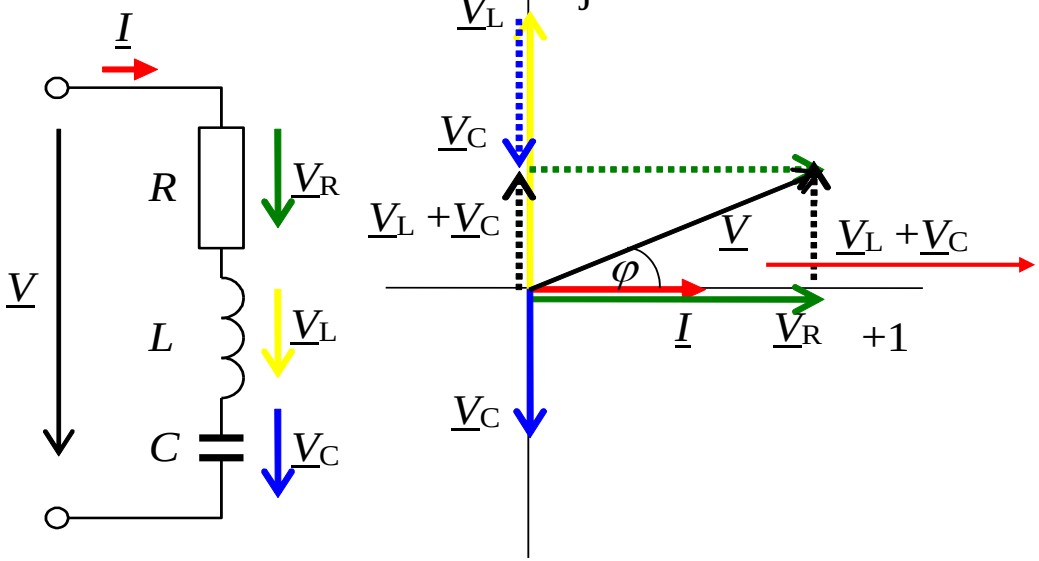
$$V_{\mathrm{m}} = \frac{1}{\omega \cdot C} \cdot I_{\mathrm{m}} = X_{\mathrm{C}} \cdot I_{\mathrm{m}} \rightarrow X_{\mathrm{C}} = \frac{1}{\omega \cdot C} \quad (\Omega)$$

$$V = X_{\mathrm{C}} \cdot I$$

$$\varphi = 90^{0} \rightarrow \cos\varphi = 0 \rightarrow P = 0$$

$$\sin\varphi = 1 \rightarrow Q = V \cdot I = X_{\mathrm{C}} \cdot I^{2}$$

Series and parallel AC circuits



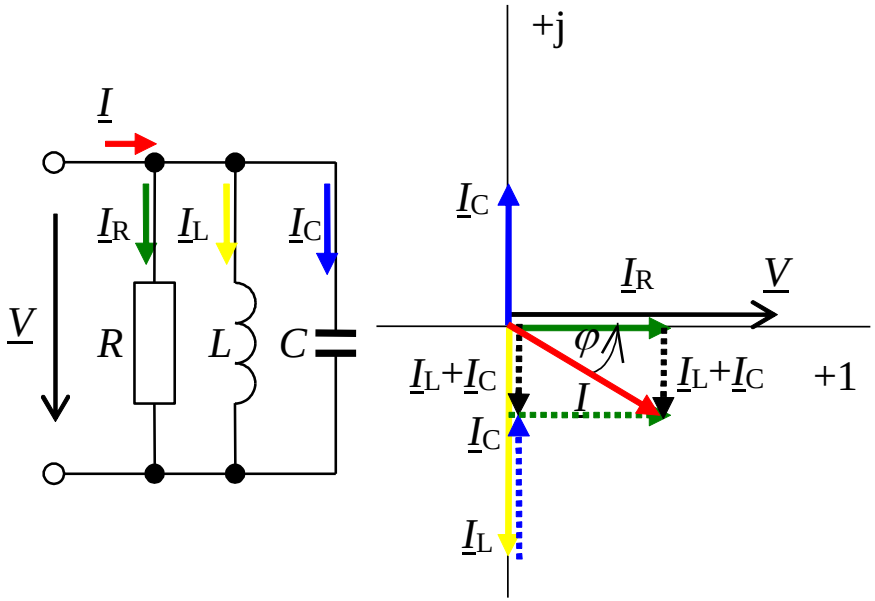
Pythagorean Theorem

$$\underline{V} = \underline{V_R} + \underline{V_L} + \underline{V_C}$$

$$V = \sqrt{V_R{}^2 + (V_L - V_C)^2}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$\varphi = \operatorname{arctg}\frac{V_L - V_C}{V_R} = \operatorname{arctg}\frac{X_L - X_C}{R}$$



$$\underline{I} = \underline{I_R} + \underline{I_L} + \underline{I_C}$$

$$I = \sqrt{I_R{}^2 + (I_L - I_C)^2}$$

$$\frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

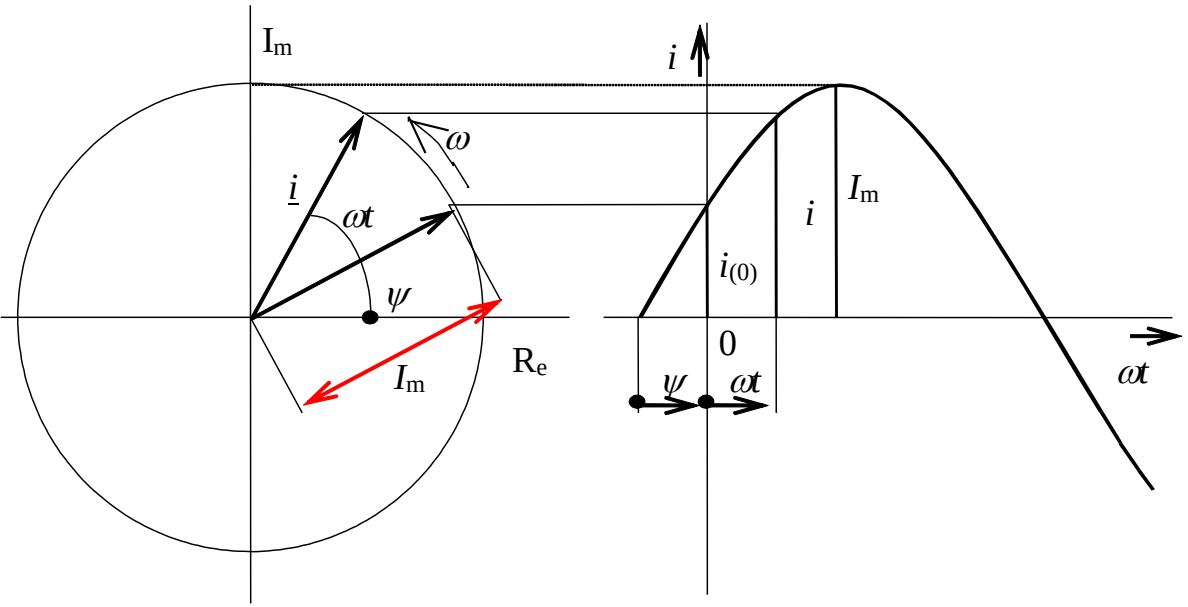
$$\varphi = \operatorname{arctg}\frac{I_L - I_C}{I_R}$$

$$\frac{1}{Z} = Y$$

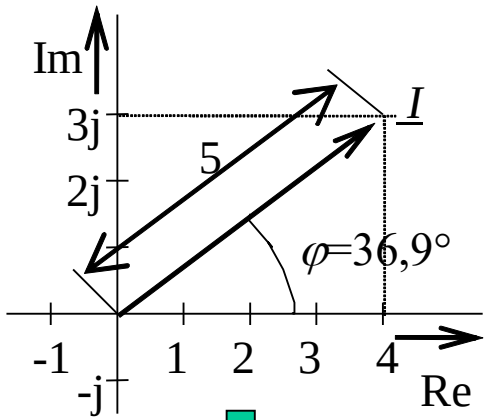
More complex AC circuits

Phasor solution (representing sinusoidal signals by complex number)

Harmonic current as the phasor



Current rms as the complex number

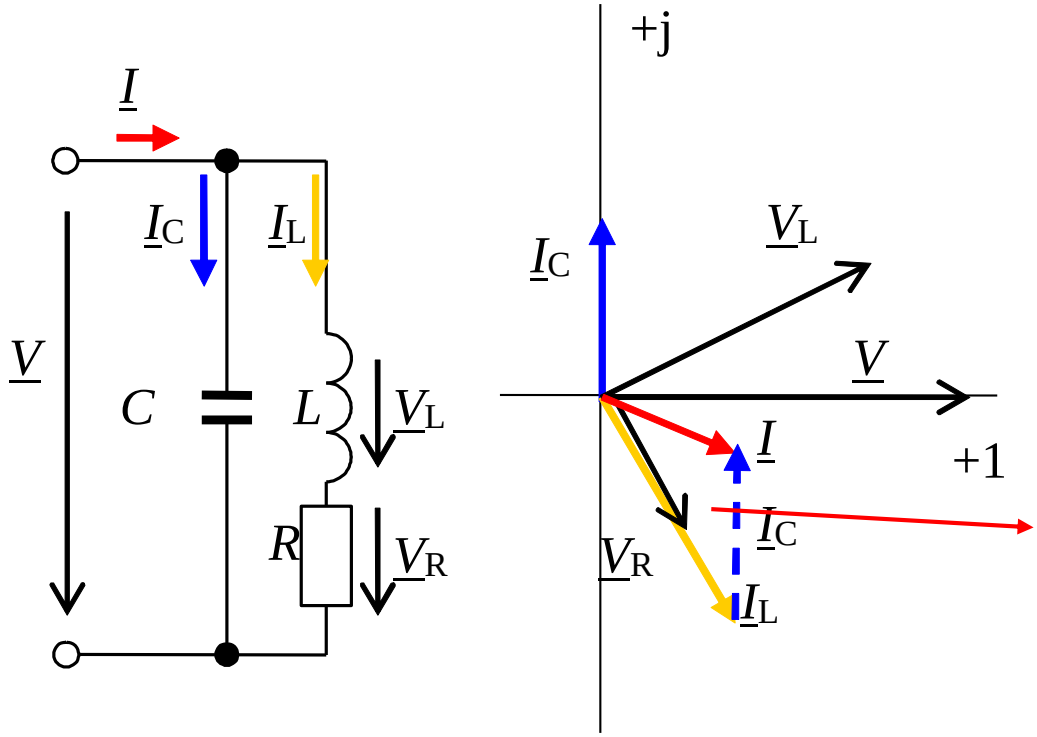


$$\underline{I} = \mathrm{Re}(\underline{I}) + \mathrm{Im}(\underline{I}) = I_x + \mathrm{j}\, I_y$$

$$I_x = I \cdot \cos\varphi\,,\quad I_y = I \cdot \sin\varphi$$

$$\underline{I} = I \cdot \cos\varphi + \mathrm{j} I \cdot \sin\varphi \rightarrow \underline{I} = I \cdot \mathrm{e}^{\mathrm{j}\varphi}$$

More complex AC circuits



no rectangular triangle

Phasor solution

$$\underline{R} = R$$

$$\underline{X_{\mathrm{L}}} = \mathrm{j}X_{\mathrm{L}}$$

$$\underline{X_{\mathrm{C}}} = -\mathrm{j}X_{\mathrm{C}}$$

$$\underline{I} = \underline{I_{\mathrm{C}}} + \underline{I_{\mathrm{L}}} = \frac{\underline{V}}{-\mathrm{j}X_{\mathrm{C}}} + \frac{\underline{V}}{R + \mathrm{j}X_{\mathrm{L}}}$$

$$\frac{1}{\underline{Z}} = \underline{Y} = \frac{1}{-\mathrm{j}X_{\mathrm{C}}} + \frac{1}{R + \mathrm{j}X_{\mathrm{L}}} = \mathrm{j}\frac{1}{X_{\mathrm{C}}} + \frac{1}{R + \mathrm{j}X_{\mathrm{L}}}$$